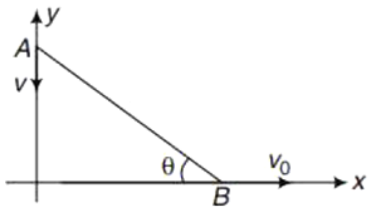
PHYSICS

1.(AB) Velocity component along $AB = 0$



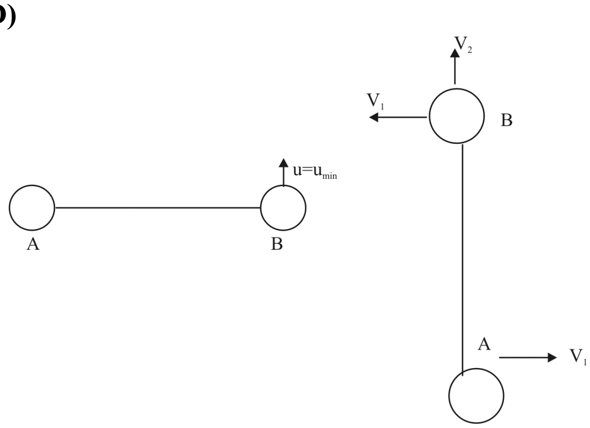
$\therefore \quad v_0 \cos\theta = v\sin\theta$ or $v = v_0 \cot\theta = f_1(\theta)$

At $\theta = 37°, v = \frac{4}{3}v_0$

$$\omega = \frac{\text{Relative velocity} \perp \text{to } AB}{l} = \frac{v_0 \sin\theta + v\cos\theta}{l} = f_2(\theta) = \frac{v_0(3/5) + (4/3v_0)(4/5)}{l} = \frac{5v_0}{3l}$$

2.(AD) As capacitance of system increases so charge on C_1 increases and, charge on C_2 decreases as voltage across it decreases.

3.(ABD)



Centre of mass does not move horizontally.
C.O.L.M along horizontal direction is valid.

C.O.E: $\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}m\left(v_1^2 + v_2^2\right) + mgl$

$\Rightarrow \quad u^2 = 2v_1^2 + v_2^2 + 2gl \qquad \ldots\ldots \text{(i)}$

For just lift off, $v_2 = 0 \Rightarrow u^2 = 2v_1^2 + 2gl$

From F.B.D of A w.r.t ground (at just lift off),

$T = mg \qquad \ldots\ldots \text{(ii)}$

Centre of mass at just lift off, has acceleration $= g(downward) \quad \because \quad N = 0$

From F.B.D of B w.r.t centre of mass,

$T = m\omega^2 \frac{l}{2} \qquad \ldots\ldots \text{(iii)}$

$\Rightarrow \quad \omega\,(\text{at lift off}) = \dfrac{2v_1}{l}$ ...... (iv)

$\Rightarrow \quad mg = m\omega^2 \dfrac{l}{2} \quad \Rightarrow \quad 2g = \dfrac{4v_1^2}{l^2}\cdot l \quad \Rightarrow \quad v_1^2 = g\dfrac{l}{2}$

From (i),

$$u_{\min}^2 = 2v_1^2 + 2gl = 3gl \Rightarrow u_{\min} = \sqrt{3gl}$$

For $u = \sqrt{4gl}$, A will leave before $\theta = 90°$

For $u = \sqrt{2gl}$, A will not leave.

For finding minimum u so that rope just slacks at top location:

$$u^2 = 2v_1^2 + v_2^2 + 2gl$$

$$v_2 \approx 0$$

$$u^2 - 2gl = 2v_1^2$$

For $T = 0$, at top-location,

$\because$ Acceleration of centre of mass of $A + B = \dfrac{g}{2}$ downwards

$\because \quad \omega = \dfrac{2v_1}{l}$

From F.B.D of B, w.r.t. C.O.M.

$$mg - \frac{mg}{2} = m\omega^2 \frac{l}{2}$$

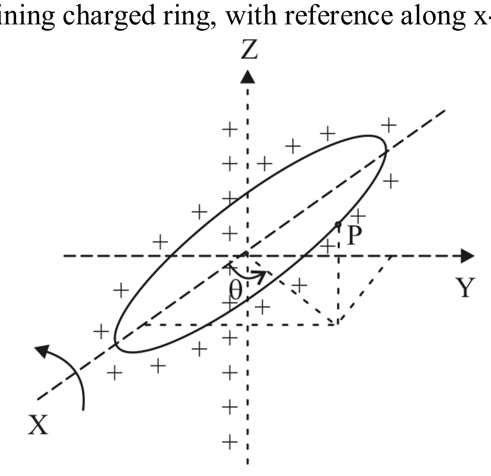
$\Rightarrow \quad \dfrac{mg}{2} = m\dfrac{l}{2}\dfrac{4v_1^2}{l^2} \quad \Rightarrow \quad v_1^2 = \dfrac{gl}{4}$

So, $u^2 - 2gl = 2v_1^2 = \dfrac{gl}{2}$

$\Rightarrow \quad u^2 = 2.5gl \quad \Rightarrow \quad u = \sqrt{2.5gl}$

4.(ACD)

Position vector of point $P = R\left(\cos\phi\hat{i} + \sin\phi.\cos\alpha\hat{j} + \sin\phi.\sin\alpha\hat{k}\right)$ where ϕ is polar angle on the plane containing charged ring, with reference along x-axis.



Perpendicular distance of point p from charged wire

$$= r_{\perp} = \sqrt{x^2 + y^2} = R\sqrt{\cos^2\phi + \sin^2\phi.\cos^2\alpha}$$

Let polar angle for foot of perpendicular of point P on xy plane (i.e. point P_{xy}) be θ

Then $\tan\theta = \frac{y}{x} = \frac{\sin\phi\cos\alpha}{\cos\phi}$

i.e., $\tan\theta = \tan\phi\cos\alpha$ …… (i)

Electric field due to charged ring at point P $= \frac{\lambda_1}{2\pi\in_0 r_{\perp}}\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right) = \vec{E}$

Force on charge at point P $= \lambda_2 Rd\phi\vec{E} = d\vec{F}$

i.e., $d\vec{F} = \frac{\lambda_1\lambda_2 Rd\phi}{2\pi\in_0 r_{\perp}}\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$

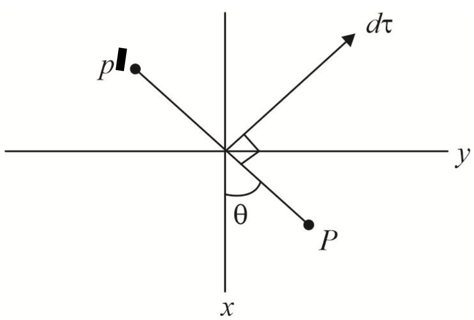
For each point P on ring in $(+x,+y)$ there exist a point p' on ring in $(-x,-y)$ where small force on p' is counter-balancing force on P. Similarly, we can say about point S in $(-x,+y)$, whose conjugate exist(s') in $(+x,-y)$

So, $\vec{F}_{net}$ (on ring) due to wire $= 0$

But torque about origin is not zero.

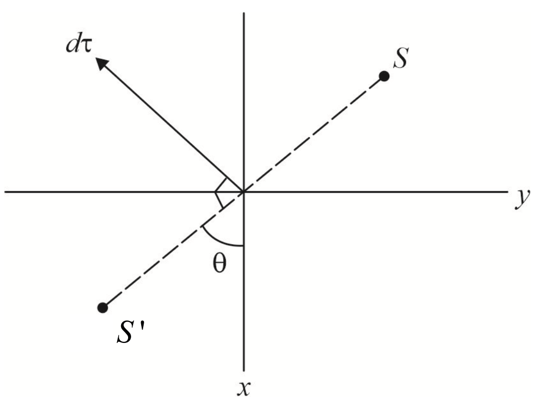
Torque due to forces on P and p'

$d\tau = dF.(2z)$ (direction as shown)



Torque due to forces on s and s'

$d\tau = dF.(2z)$(direction as shown)



Net torque on ring (by symmetry) is along –x axis

$$\vec{\tau} = -\int_{\phi=0}^{2\pi} d\tau \cos(90-\theta)\hat{i} \;= -\int_{\phi=0}^{2\pi} d\tau \sin\theta \hat{i}$$

Finding, $\displaystyle\int_{\phi=0}^{2\pi} d\tau \sin\theta = \int_{\phi=0}^{2\pi} \frac{\lambda_1\lambda_2 R d\phi}{2\pi \in_0 r_\perp} 2z\sin\theta$

$$= \int_{\phi=0}^{2\pi} \frac{\lambda_1\lambda_2 R d\phi}{\in_0 \pi R\sqrt{\cos^2\phi+\sin^2\phi\cos^2\alpha}} 2R\sin\phi\sin\alpha\sin\theta$$

$$= \frac{2\lambda_1\lambda_2 R}{\pi\in_0}\int_{\phi=0}^{2\pi} \frac{\sin\phi\sin\alpha\sin\theta d\phi}{\sqrt{\cos^2\phi+\sin^2\phi\cos^2\alpha}} = \frac{2\lambda_1\lambda_2 R}{\pi\in_0}\int_{\phi=0}^{2\pi}\frac{\tan\phi\sin\theta d\phi.\sin\alpha}{\sqrt{1+\tan^2\phi\cos^2\alpha}}$$

$\because \quad \tan\theta = \tan\phi\cos\alpha$ $\displaystyle = \frac{2\lambda_1\lambda_2 R}{\pi\in_0}\int_{\phi=0}^{2\pi}\frac{\tan\phi\sin\theta\sin\alpha d\phi}{\sec\theta}$

$$= \frac{2\lambda_1\lambda_2 R\sin\alpha}{\pi\in_0}\int_{\phi=0}^{2\pi}\tan\phi\sin\theta\cos\theta d\phi$$

$$= \frac{2\lambda_1\lambda_2 R\sin\alpha}{\pi\in_0}\int_{\phi=0}^{2\pi}\frac{\tan\theta}{\cos\alpha}\sin\theta\cos\theta d\phi$$

$$= \frac{2\lambda_1\lambda_2 \tan\alpha.R}{\pi\in_0}\int_{\phi=0}^{2\pi}\sin^2\theta d\phi = \frac{2\lambda_1\lambda_2\tan\alpha.R}{\pi\in_0}\int_{\theta=0}^{2\pi}\frac{\sin^2\theta\sec^2\theta d\theta}{\sec^2\phi.\cos\alpha}$$

$$= \frac{2\lambda_1\lambda_2\sin\alpha.R}{\pi\in_0\cos^2\alpha}\int_{\theta=0}^{2\pi}\frac{\tan^2\theta d\theta}{1+\sec^2\alpha\tan^2\theta}$$

$$= \frac{8\lambda_1\lambda_2\sin\alpha.R}{\pi\in_0\cos^2\alpha}\int_{\theta=0}^{90^\circ}\frac{\tan^2\theta d\theta}{1+\sec^2\alpha\tan^2\theta} = \frac{8\lambda_1\lambda_2 R\sin\alpha}{\pi\in_0\cos^2\alpha}.I$$

Where $\displaystyle I = \int_{\theta=0}^{90^\circ}\frac{\tan^2\theta}{1+K^2\tan^2\theta}d\theta$ where $k = \sec\alpha$

Let, $x = \tan\theta \quad \Rightarrow \quad dx = \sec^2\theta d\theta \quad \Rightarrow \quad d\theta = \dfrac{dx}{1+x^2}$

$$I = \int_{x=0}^{\infty}\frac{x^2}{1+K^2x^2}.\frac{dx}{1+x^2} = \frac{1}{(K^2-1)}\int_0^\infty\frac{(1+K^2x^2)-(1+x^2)}{(1+K^2x^2)(1+x^2)}dx$$

$$= \frac{1}{(K^2-1)}\left[\int_0^\infty\frac{1}{(1+x^2)}dx - \int_0^\infty\frac{1}{(1+k^2x^2)}dx\right]$$

$$= \frac{1}{(K^2-1)}\left[\frac{\pi}{2}-\frac{\pi}{2K}\right] = \frac{\pi}{2}\frac{(K-1)}{K(K^2-1)} = \frac{\pi}{2K(K+1)} = \frac{\pi}{2\sec\alpha(1+\sec\alpha)} = \frac{\pi\cos^2\alpha}{2(1+\cos\alpha)}$$

$$\tau = \frac{8\lambda_1\lambda_2 R\sin\alpha}{\pi\epsilon_0\cos^2\alpha}\times\frac{\pi}{2}\frac{\cos^2\alpha}{(1+\cos\alpha)} = \frac{4\lambda_1\lambda_2 R}{\epsilon_0}\times\frac{\sin\alpha}{(1+\cos\alpha)} = \frac{4\lambda_1\lambda_2 R}{\epsilon_0}\times\sqrt{\frac{(1-\cos\alpha)}{(1+\cos\alpha)}}$$

$$= \frac{4\lambda_1\lambda_2 R}{\epsilon_0}\times\tan\left(\frac{\alpha}{2}\right); \qquad \vec{\tau} = torque = -\frac{4\lambda_1\lambda_2 R}{\epsilon_0}\tan\left(\frac{\alpha}{2}\right)\hat{i}$$

5.(AC) $q_1 = 2\times 2 = 4\mu C$; $\qquad q_2 = 4\times 2 = 8\mu C$

$\therefore \quad \Delta q = 8-4 = 4\mu C \qquad \therefore \quad W_4 = 4\times 4 = 16\mu j$

$U_i = \frac{1}{2}\times 2\times(2)^2 = 4\ \mu J$; $\qquad U_f = \frac{1}{2}\times 2\times(4)^2 = 16\ \mu J$

$\therefore \quad \Delta U = 12\mu J \qquad \therefore \quad$ Heat $= 16-12 = 4\mu J$

6.(AC) Due to electromagnetic shielding force on inner charge = 0

Charge on inner surface of shell $= -q$

So, on outer surface charge $= Q+q$

7.(ABD)

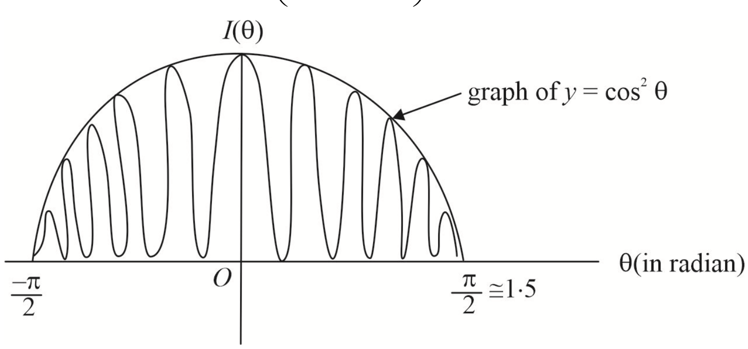
$$I(\theta) = I_1(\theta)+I_2(\theta)+2\sqrt{I_1(\theta).I_2(\theta)}.\cos\left(\frac{2\pi}{\lambda}.2d\sin\theta\right) = 4.\frac{P}{4\pi r^2}.\cos^2\left(\frac{2\pi d\sin\theta}{\lambda}\right)$$

$\because \quad I_1(\theta)\approx I_2(\theta) = \frac{P}{4\pi r^2}$

$\Rightarrow \quad I(\theta) = \frac{P}{\pi r^2}.\cos^2\left(\frac{2\pi d\sin\theta}{\lambda}\right) = \frac{p}{\pi D^2}.\cos^2\theta.\cos^2\left(\frac{2\pi d\sin\theta}{\lambda}\right)$

$\Rightarrow \quad$ Intensity at $(\theta=0)$ i.e,. $I(\theta=0) = I_0 = \frac{P}{\pi D^2}$

$$I(\theta) = I_0\cos^2\theta\cos^2\left(\frac{2\pi d\sin\theta}{\lambda}\right)$$



For maxima locations :

$$\frac{2\pi d\sin\theta}{\lambda} = \pm n\pi \ (n = 0,1,2....) \quad \Rightarrow \quad \sin\theta = \pm\frac{n\lambda}{2d} = \pm\frac{n}{200} = \left(0,\pm\frac{1}{200},\pm\frac{2}{200},....\right)$$

Where $n = 200$, $\sin\theta = \pm 1, \theta = \pm\frac{\pi}{2}$, Intensity becomes zero, due to $\cos^2\theta$.

So, for $n = 199, \sin\theta = \pm\frac{199}{200}$, *i.e., at* $\theta = \pm\sin^{-1}\left(\frac{199}{200}\right)$, we have faintest maxima.

Number of maximum $=(199\times 2)+1=399$

If screen is curved with radius equal to D about location of light sources, then expression for intensity on screen becomes.

$$I(\theta)=I_0\cos^2\left(\frac{2\pi d\sin\theta}{\lambda}\right)$$

Location of maxima $(-90°\le\theta\le 90°)$

$$\frac{2\pi d\sin\theta}{\lambda}=\pm n\pi \quad\Rightarrow\quad \sin\theta=\pm\frac{n}{200}=\left(0,\pm\frac{1}{200},\pm\frac{2}{200},.....\right)$$

All maxima will have equal intensity no. of maxima $=2\times 200+1=401$

8.(AC) $a=\frac{F}{m}$; $\quad F\frac{l}{2}\sin 60°=mg\frac{l}{2}\cos 60°$

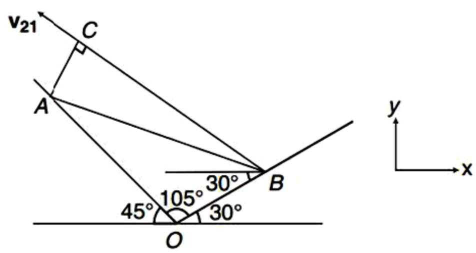
$$\frac{10\sqrt{3}}{2}=\frac{mg}{2}\quad\Rightarrow\quad m=3\ \text{kg}$$

9.(250)

$$|v_{21x}|=(v_1+v_2)\cos 60°=12\,m/s$$

$$|v_{21y}|=(v_2-v_1)\sin 60°=4\sqrt{3}\,m/s$$

$$\therefore\ v_{21}=\sqrt{(12)^2+(4\sqrt{3})^2}=\sqrt{192}\ m/s$$

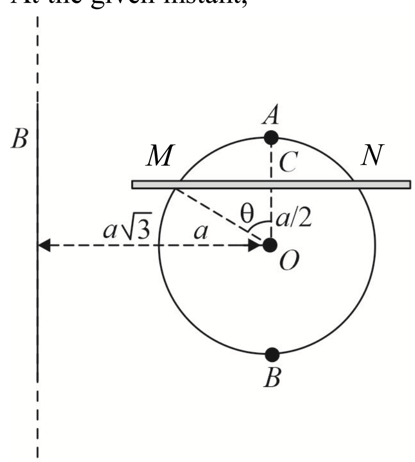


$BC=(v_{21})t=240\,m$; $\quad AC=70m$ (given)

Hence, $AB=\sqrt{(240)^2+(70)^2}=250\,m$

10.(0.34)

(A) At the given instant,



$AC = \frac{a}{2}, OC = \frac{a}{2}$ and $\cos\theta = \frac{a/2}{a} = \frac{1}{2}, \theta = \frac{\pi}{3}$

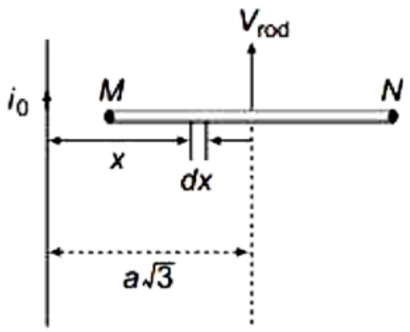
$\therefore$ Velocity of rod $= \left(\frac{+v_0}{2}\right)$ along the direction of current

Emf induced across the ends M and N

$$E_{rod} = \int_{a\sqrt{3}-\frac{a\sqrt{3}}{2}}^{a\sqrt{3}+\frac{a\sqrt{3}}{2}} v_{rod} B dx = v_{rod} \int_{-\frac{a\sqrt{3}}{2}}^{3a\frac{\sqrt{3}}{2}} \frac{\mu_0 i_0}{2\pi x} dx = \left(\frac{v_0}{2}\right)\frac{\mu_0 i_0}{2\pi} \ln\left(\frac{3}{1}\right)$$

With end M at higher potential.

Since, the effective length of both the arcs MAN and MBN is MN.



$$\therefore \qquad E_{MAN} = E_{MBN} = v_{loop} \frac{\mu_0 i_0}{2\pi} \ln 3 = v_0 \frac{\mu_0 i_0}{2\pi} \ln 3$$

With point M at higher potential .

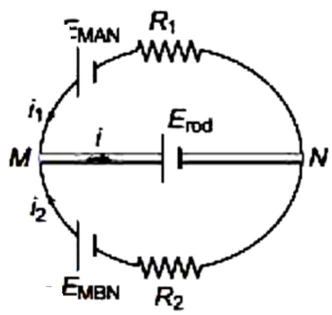
Resistance of arc MAN

$\Rightarrow \qquad R_1 = (R)2(a\theta) = 2aR\frac{\pi}{3}$

$\Rightarrow \qquad$ Resistance of arc MBN

$\Rightarrow \qquad R_2 = (R)(a)(2\pi - 2\theta) = 4aR\frac{\pi}{3}$

Equivalent circuit at the given instant is shown in the figure.

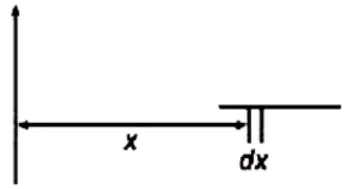


Current through the rod MN,

$$i = (i_1 + i_2) = \left(\frac{E_{MAN} - E_{rod}}{R_1}\right) + \left(\frac{E_{MBN} - R_{rod}}{R_2}\right)$$

$$i=\left(E_{MAN}-E_{rod}\right)\left[\frac{1}{R_1}+\frac{1}{R_2}\right]=\frac{v_0\mu_0 i_0(\ln 3)}{4\pi}\left[\left(\frac{1}{2}+\frac{1}{4}\right)\frac{3}{aR\pi}\right]=\frac{9v_0\mu_0 i_0(\ln 3)}{16aR\pi^2}$$

Force on the rod



$$F_{rod}=\int_{\frac{a\sqrt{3}}{2}}^{\frac{3a\sqrt{3}}{2}} idxB=\frac{i\mu_0 i_0}{2\pi}\ln 3=\frac{9i_0^2\mu_0^2 v_0}{32aR\pi^3}(\ln 3)^2\cong .34\frac{i_0^2\mu_0^2 v_0}{aR\pi^3}$$

11.(7) $\Omega_1=2\pi\left(1-\frac{1}{2}\right)=\pi$

$$\Omega_2=2\pi\left(1-\frac{a}{\sqrt{a^2+h^2}}\right)\qquad\therefore\qquad \phi_1=\phi_2$$

$$\Rightarrow\quad \frac{\pi}{4\pi}\times\left(\frac{q}{\varepsilon_0}\right)=\frac{2\times 2\pi}{4\pi}\left(1-\frac{a}{\sqrt{a^2+h^2}}\right)\times\left(\frac{q}{\varepsilon_0}\right)$$

$$\Rightarrow\quad \frac{1}{4}=1-\frac{a}{\sqrt{a^2+h^2}}\quad\Rightarrow\quad h=\frac{\sqrt{7}}{3}a\quad\Rightarrow\quad n=7$$

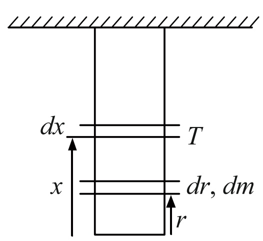
12.(1.61) $E=\frac{12400}{4000}=3.1eV$

Number of photoelectrons emitted per second,

$$n=\left(\frac{1}{10^6}\right)\left(\frac{5}{3.1\times 1.6\times 10^{-19}}\right)=1.0\times 10^{13}\text{ per second}$$

$$\therefore\qquad i=ne=1.0\times 10^{13}\times 1.6\times 10^{-19}=1.6\times 10^{-6}A=1.61\mu A$$

13.(60) $T=\int_0^x dmg=\int_0^x \lambda_0\left(1+\frac{r}{\ell}\right)dr.g=\lambda_0 g\left(x+\frac{x^2}{2\ell}\right)$



$T\rightarrow$ tension in thickness dx

$dy\rightarrow$ elongation in thickness dx

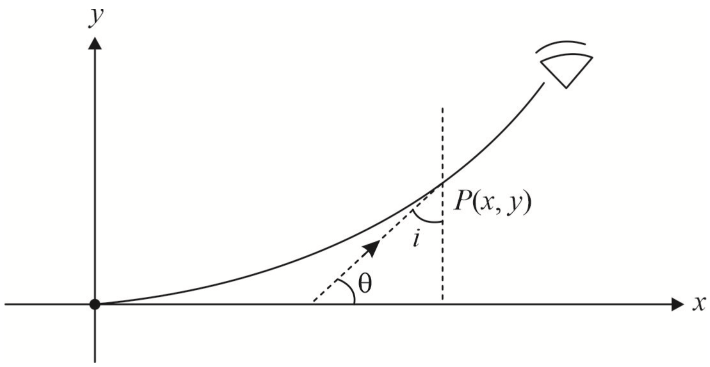
$dU\rightarrow$ energy stored in dx

$$dy=\frac{Tdx}{YA};\qquad dU=\frac{1}{2}Tdy$$

$$\int dU=\int_0^\ell \frac{T^2dx}{2AY}$$

$$U=\frac{\lambda_0^2 g^2}{2AY}\int_0^\ell\left(x^2+\frac{x^4}{4\ell^2}+\frac{x^3}{\ell}\right)dx=\frac{19\lambda_0^2 g^2\ell^2}{60AY}$$

14.(2) $\theta = 90° - i$



$\tan\theta = \cot i$

or $\dfrac{dy}{dx} = \cot i$ ...... (i)

$\mu_0 \sin i_0 = \mu_p \sin i_p$

(i) $\sin 90° = \left(\sqrt{1+\alpha y}\right)\sin i$

$\therefore \quad \sin i = \dfrac{1}{\sqrt{1+\alpha y}}; \ \cot i = \sqrt{\alpha y} = \dfrac{dy}{dx}$

$\therefore \quad \int_0^y \dfrac{dy}{\sqrt{\alpha y}} = \int_0^x dx \ \ or \ \ x = 2\sqrt{\dfrac{y}{a}}$

Substituting $y = 2m, and\ a = 2.0\times10^{-6} m^{-1}$

We get $x_{\max} = 2000m = 2\,km$

15.(C) Highest fundamental frequency will occur at length $= l_0$

$$f_1 = \frac{1}{2l_0}\sqrt{\frac{T_0}{\mu}} = f_0;\ f_2 = \frac{1}{2l_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2};\ \ f_3 = \frac{1}{2l_0}\sqrt{\frac{T_0}{9\mu}} = \frac{f_0}{3};\ f_4 = \frac{1}{2l_0}\sqrt{\frac{T_0}{16\mu}} = \frac{f_0}{4}$$

16.(D) Fundamental frequency of string -1 $= \dfrac{1}{2l_0}\sqrt{\dfrac{T_0}{\mu}} = f_0$ $\qquad f_1 = 1^{st}$ harmonic $= f_0$

Fundamental frequency of string -2 $= \dfrac{1}{4l_0}\sqrt{\dfrac{T_0}{4\mu}} = \dfrac{f_0}{4}$ $\qquad f_2 = 2^{nd}$ harmonic $= \dfrac{f_0}{2}$

Fundamental frequency of string -3 $= \dfrac{1}{24l_0}\sqrt{\dfrac{T_0}{9\mu}} = \dfrac{f_0}{36}$ $\qquad f_3 = 6^{th}$ harmonic $= \dfrac{f_0}{6}$

Fundamental frequency of string -4 $= \dfrac{1}{16l_0}\sqrt{\dfrac{T_0}{16\mu}} = \dfrac{f_0}{32}$ $\qquad f_4 = 4^{th}$ harmonic $= \dfrac{f_0}{8}$

17.(C) Work done in $(1\to2\to3)$ = Area under graph $= 2P_0V_0 = 2RT_0$

Change in internal energy in $(1\to2\to3) = \dfrac{5R\Delta T}{2} = \dfrac{5}{2}R(T_3 - T_1) = \dfrac{5}{2}(P_3V_3 - P_1V_1)$

$$= \frac{5}{2}\left(4P_0V_0 - P_0V_0\right) = \frac{15}{2}P_0V_0 = \frac{15}{2}RT_0$$

Heat absorbed in $\left(1 \to 2 \to 3\right) = \Delta U + W = \frac{15}{2}RT_0 + \frac{4}{2}RT_0 = \frac{19}{2}RT_0$

Heat absorbed in $\left(1 \to 2\right) = \Delta U_{1\to 2} + W_{1\to 2} = \Delta U_{1\to 2} = \frac{5}{2}R\left(T_2 - T_1\right) = \frac{5}{2}P_0V_0 = \frac{5}{2}RT_0$

18.(D) Work done in $\left(1 \to 2\right) = P_0\left(V_2 - V_1\right) = RT_2 - RT_1 = R\left(T_2 - T_1\right) = RT_0$

Work done in $\left(2 \to 3\right) = R\left(2T_0\right)\log_e\left(\frac{P_2}{P_3}\right) = -2RT_0\log_e 2$

Work done in $\left(1 \to 2 \to 3\right) = RT_0\left(1 - 2\log_e 2\right) = -RT_0\left(\ln 4 - 1\right)$

Change in internal energy $\left(1 \to 2\right) = \frac{5R}{2}\left(T_2 - T_1\right) = \frac{5}{2}RT_0$

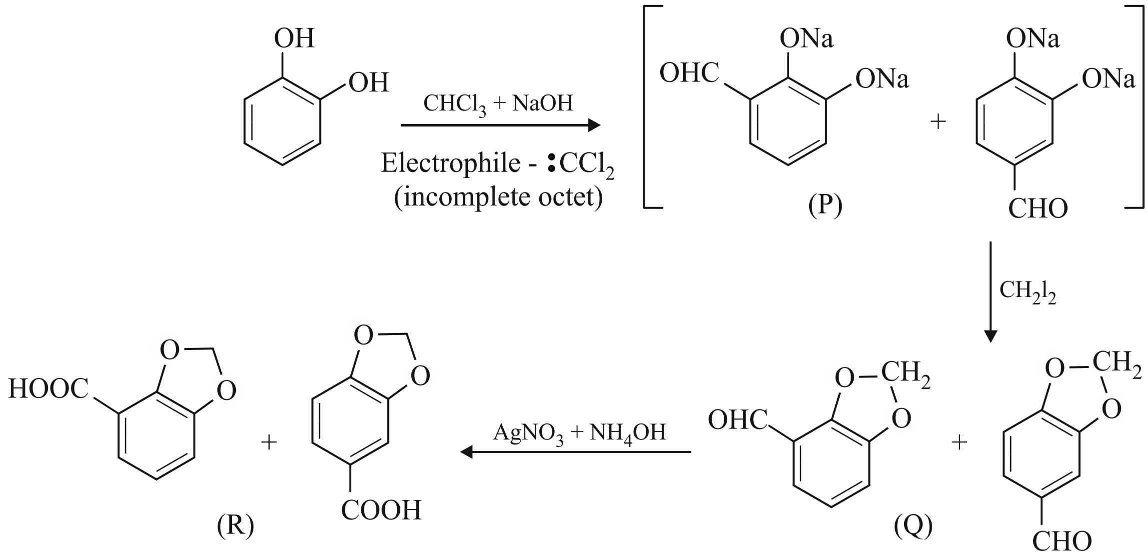
Change in internal energy $\left(1 \to 2 \to 3\right) = \frac{5R}{2}\left(T_3 - T_1\right) = \frac{5}{2}RT_0$

$\Delta Q\left(for\,1 \to 2 \to 3\right) = \Delta U + W = \frac{5}{2}RT_0 - RT_0\left(\ln 4 - 1\right) = RT_0\left(\frac{7}{2} - \ln 4\right)$

$\Delta Q\left(for\,1 \to 2\right) = \Delta U + W = \frac{5}{2}RT_0 + RT_0 = \frac{7}{2}RT_0$

CHEMISTRY

1.(ABD)



2.(AC)

(A) $CO_2 \xrightarrow{Ca(OH)_2} CaCO_3\downarrow$ (Milkiness)

$CO_2 \xrightarrow{MnO_4^-/H^+}$ No reaction

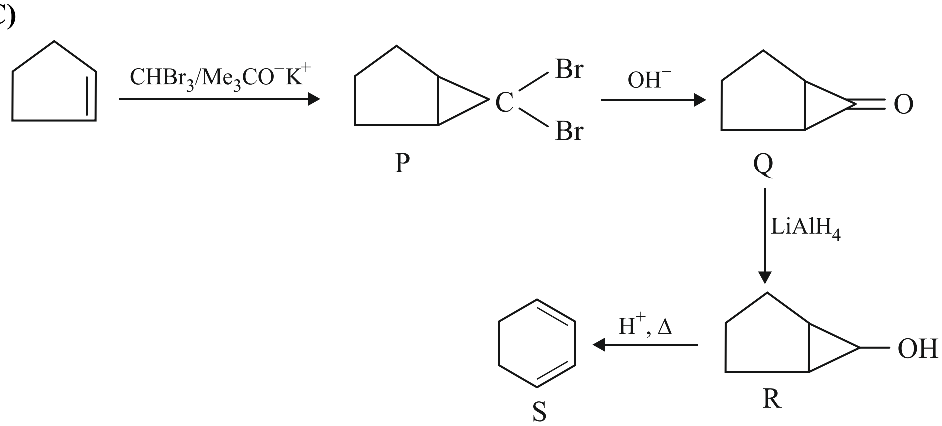
(B) $SO_2 \xrightarrow{Cr_2O_7^{2-}/H^+} Cr^{3+} + SO_4^{2-}$ (Green)

$SO_2 \xrightarrow{NaOH} SO_3^{2-} \xrightarrow{BaCl_2} BaSO_3\downarrow$ (White ppt)

(C) $2NO_2 \xrightarrow{\text{Cooled to } -11°C} N_2O_4$ (Colorless)

$2NO_2 \xrightarrow[\text{(excess)}]{NaOH} NaNO_2 + NaNO_3 \xrightarrow{Zn} NH_3\uparrow$

3.(ABC)



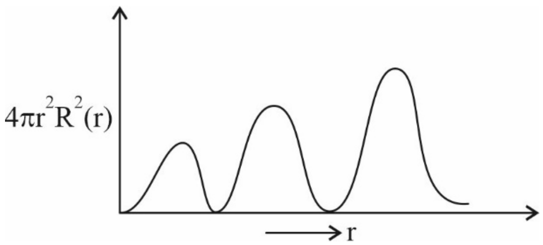
4.(ABD)

Statements A, B & D are correct. Carrier proteins carry polar molecules across the cell membrane.

5.(ACD)

(A) Orbital is 3s

(B) Curve will be



(C) For radial nodes

$$\sigma = \frac{+6 \pm \sqrt{36-24}}{2}$$

$$\frac{2r}{3a_0} = 3 \pm \sqrt{3}$$

$$r = \frac{9 \pm 3\sqrt{3}}{2} a_0$$

First radial node $\quad r = \frac{9-3\sqrt{3}}{2} a_0$

Second radial node $\quad r = \frac{9+3\sqrt{3}}{2} a_0$

Distance between radial nodes $= \frac{9+3\sqrt{3}}{2} a_0 - \frac{9-3\sqrt{3}}{2} a_0 = 3\sqrt{3}\, a_{a0}$

(D) Radial nodes $n - \ell - 1 = -3 - 0 - 1 = 2$

6.(ACD)

$P \rightarrow CaO$; $Q \rightarrow$ Blister copper; $R \rightarrow Fe$; $S \rightarrow SO_2$

7.(BCD)

KO_2 and NaO_2 both are superoxides.

(I) KO_2 is more stable than NaO_2 as K^+ is larger than Na^+

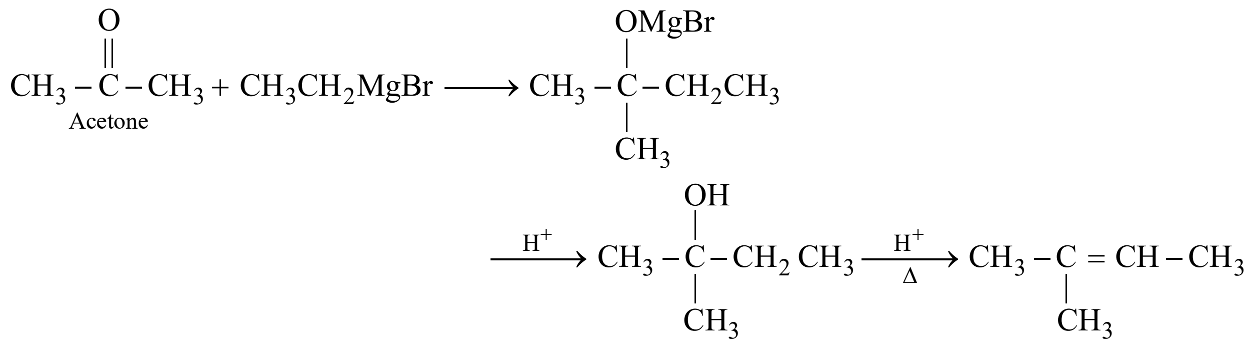
(II) Products of reaction of metal & HNO_3 depends upon conc. Of HNO_3

(III) $P_4 + NaOH \longrightarrow PH_3 + NaH_2PO_2$

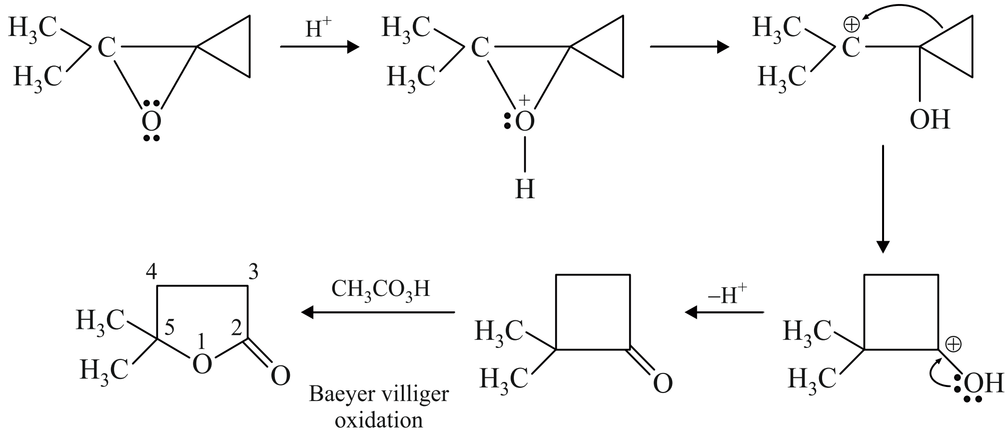
(IV) $\underset{\text{(orange)}}{(NH_4)_2Cr_2O_7} \xrightarrow{\Delta} N_2 + \underset{\substack{\text{green colored}\\ \text{compound}\\ \text{used in}\\ \text{fireworks}}}{Cr_2O_3} + H_2O$

8.(BD) $\underset{(A)}{(CH_3)_2CHCHOHCH_3} \xrightarrow[250°C]{Al_2O_3} \underset{(B)}{CH_3-\underset{\underset{CH_3}{|}}{C}=CH-CH_3} \xrightarrow[AgOH]{HI} \underset{(C)}{CH_3-\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}-CH_2-CH_3}$

$\xrightarrow{Al_2O_3} \underset{(B)}{CH_3-\underset{\underset{CH_3}{|}}{C}=CH-CH_3} \xrightarrow[(ii)\,H_2O_2,OH^-]{(i)\,B_2H_6} \underset{(A)}{(CH_3)_2CHCHOHCH_3}$



9.(5)



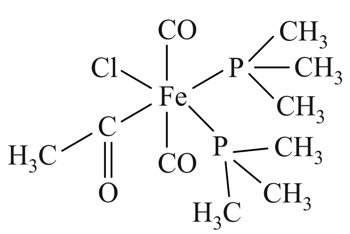
10.(183)

$$\text{Final molarity} = \frac{M_1V_1 + M_2V_2 + M_3V_3}{V_1 + V_2 + V_3}$$

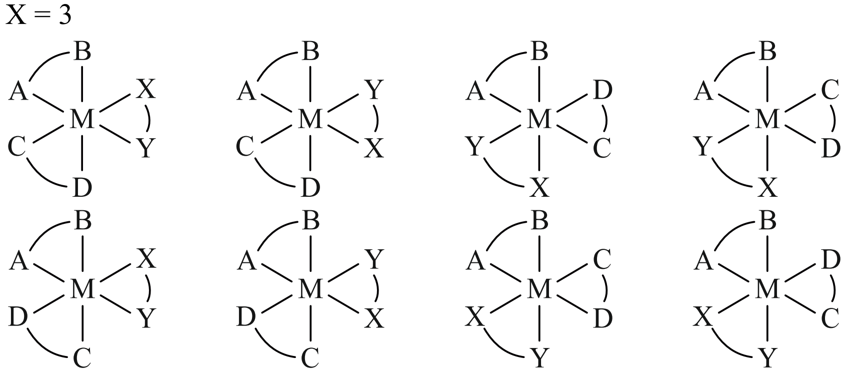
$$\text{For 10\% w/v NaOH} \quad \Rightarrow \quad \text{Molarity} = \frac{10 \times 10}{40} = 2.5\,\text{M}$$

$$\therefore \quad \text{Final molarity} = \frac{1 \times V + 2.5 \times V + 2 \times V}{3V} = 1.83\,\text{M}$$

11.(11)



X = 3



Y = 8 $\quad \Rightarrow \quad$ X + Y = 3 + 8 = 11

12.(16) $\overset{+5}{H}NO_3 \xrightarrow{Zn} \overset{+1}{N_2}O$

Change in oxidation state = 4

$$Ca_3P_2 \;+\; 6H_2O \longrightarrow 3Ca(OH)_2 \;+\; 2PH_3\uparrow$$

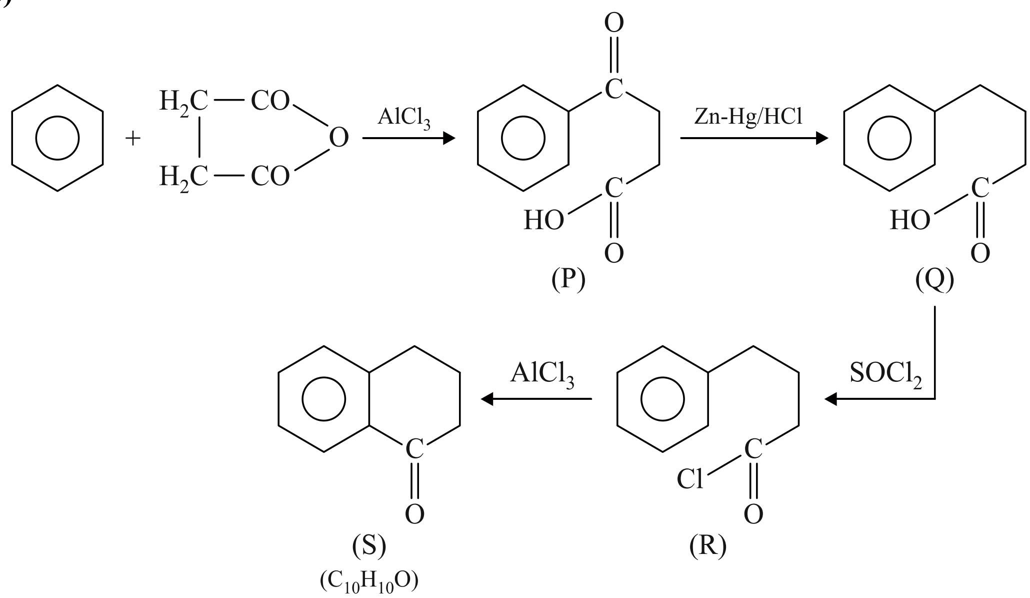
1 mol — 2 mol

2 mol — 4 mol

$A \times B = 4 \times 4 = 16$

13.(30) Half life $= \dfrac{0.693}{k_1 + k_2} = \dfrac{0.693}{2 \times 6.93} \times 10^4 \text{ min} = 500 \text{ min} = 30000 \text{ s}$

14.(146)

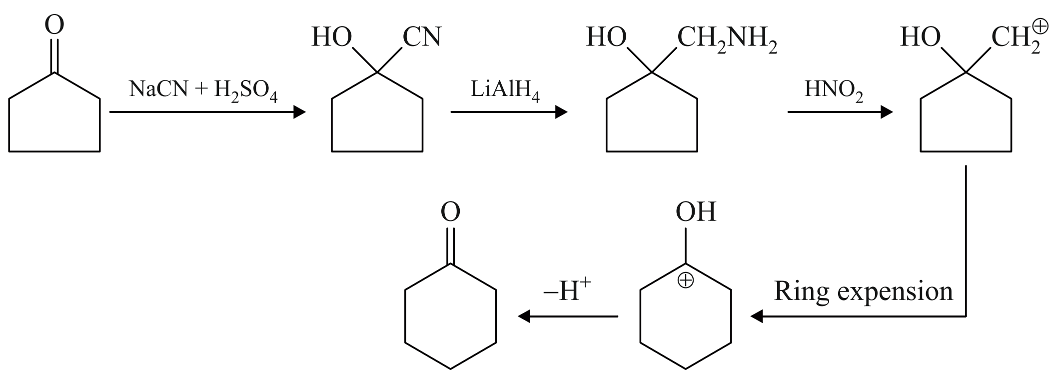


Molar mass of 'S' $= 146 \text{ g mol}^{-1}$

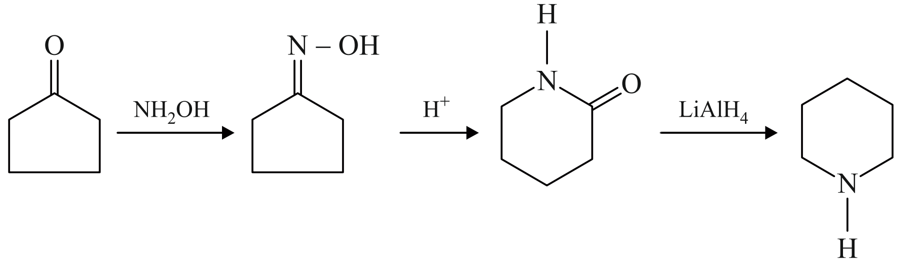
15.(A) **(I)** Number of radial nodes $= n - \ell - 1$ $\therefore$ The graph could be of 4s, 5p, 6d.

16.(D) p, d orbitals have angular nodes.

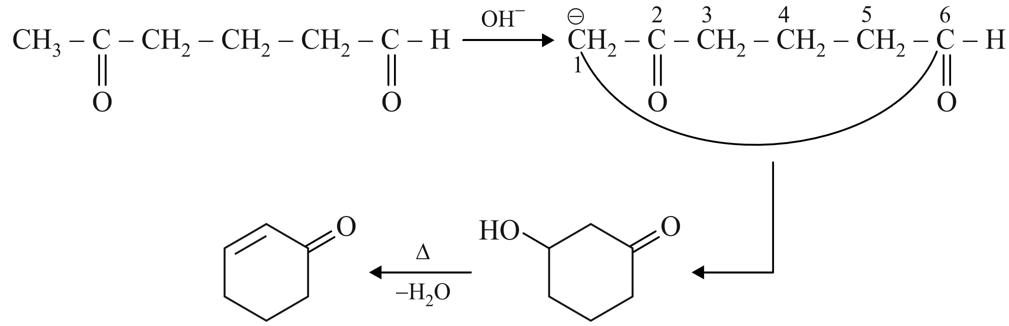
17.(C) **(I)**



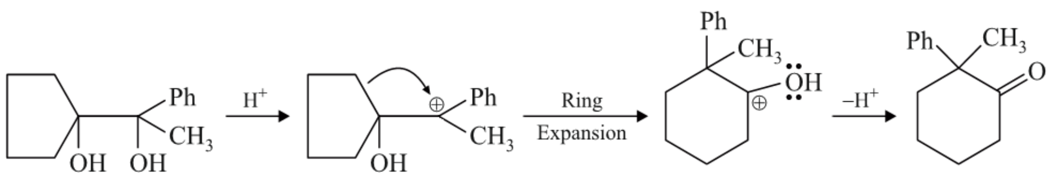
(II)



(III)



(IV)



18.(B) Solutions same as Q. 3

MATHEMATICS

1.(BD) $\left(A^2-2I\right)B=0 \Rightarrow A^2=2I$ and $B=adjA=|A|\dfrac{A}{2}=-\sqrt{2}A \quad \left(\because |A|=-2\sqrt{2}\right)$

$AB=A(adjA)=|A|.I$; $\qquad tr.(AB)=3|A|=-6\sqrt{2}$

And det. $\left(A-\sqrt{2}B\right)=\det.(A+2A)=\det.(3A)=27\left(-2\sqrt{2}\right)=-54\sqrt{2}$

2.(ABCD)

$$p^n=\begin{bmatrix}1 & n & 0\\ 0 & 1 & 0\\ 0 & n & 1\end{bmatrix} \text{ and } B^n=QP^nQ^T \quad \Rightarrow \quad \det.(A)=1 \text{ and } \det.(B)=1$$

3.(AC)

Let $3^k=x$; $\quad 2^k=y$

$$\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}=\frac{xy}{(x-y)(3x-2y)}$$

$$=\frac{y(3x-2y)-2y(x-y)}{(x-y)(3x-2y)}=\frac{y}{x-y}-\frac{2y}{3x-2y}=\frac{2^k}{3^k-2^k}-\frac{2^{k+1}}{3^{k+1}-2^{k+1}}$$

$$\therefore \quad \sum_{k=1}^{\infty}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}; \qquad \sum_{k=1}^{\infty}\frac{2^k}{\left(3^k-2^k\right)}-\frac{2^{k+1}}{\left(3^{k+1}-2^{k+1}\right)}$$

$$T_1=\frac{2}{3-2}-\frac{2^2}{3^2-2^2}$$

$$T_2=\frac{2^2}{3^2-2^2}-\frac{2^3}{3^3-2^3}$$

……

$$T_k=\frac{2^k}{3^k-2^k}-\frac{2^{k+1}}{3^{k+1}-2^{k+1}}; \qquad \sum_{k=1}^{\infty}T_k=\lim_{k\to\infty}\left(2-\frac{2.2^k}{3.3^k-2.2^k}\right)=2$$

4.(AC) $f(x)=\dfrac{\sin^{-1}\left(1-\{x\}\right)\cos^{-1}\left(1-\{x\}\right)}{\sqrt{2\{x\}}\left(1-\{x\}\right)}$

$$f(0^+)=\lim_{x\to0^+}\frac{\sin^{-1}(1-h)\cos^{-1}(1-h)}{\sqrt{2}\sqrt{h}(1-h)}=\frac{\pi}{2\sqrt{2}}\lim_{h\to0}\frac{\cos^{-1}(1-h)}{\sqrt{h}}$$

Let $\cos^{-1}(1-h)=\theta \Rightarrow 1-h=\cos\theta \Rightarrow h=1-\cos\theta$

$$\therefore l=\frac{\pi}{2}\frac{\theta}{\sqrt{2}\sqrt{1-\cos\theta}}=\frac{\pi}{2}\lim_{\theta\to0}\frac{\theta\sqrt{1+\cos\theta}}{\sqrt{2}\sin\theta}=\frac{\pi}{2}$$

$$f\left(0^-\right)=\lim_{h\to0^-}\frac{\sin^{-1}\left(1-(1-h)\right)\cos^{-1}\left(1-(1-h)\right)}{\sqrt{2(1-h)}\left(1-(1-h)\right)}=\lim_{h\to0^-}\frac{\sin^{-1}h.\cos^{-1}h}{\sqrt{2h}}=\frac{\pi}{2\sqrt{2}}$$

5.(ABC)

$f(3-x)=f(3+x)$

$\Rightarrow$ symmetric about $x=3$

$f(x_1)=f(x_2)=f(x_3)=f(x_4)=f(x_5)=0$

$\Rightarrow x_3=3$

$x_1+x_2+x_3+x_4+x_5=15 \qquad \Rightarrow f'(3)=0$

6.(ABD)

$$I=\int_0^{\theta}\frac{2x\,dx}{\sqrt{(3\theta-2x)(\theta+2x)}}=\int_0^{\theta}\frac{2(\theta-x)\,dx}{\sqrt{(3\theta-2x)(\theta+2x)}} \quad \Rightarrow \quad I=\frac{\theta}{2}\int_0^{\theta}\frac{dx}{\sqrt{\theta^2-(x-\theta/2)^2}}=\frac{\theta\pi}{6}$$

7.(AC) Has local maximum at $x=0$

$\Rightarrow \quad f(x)=c-ax^2,\ c,\ a\in Q$ and $a>0$

$f'(x)=-2ax$

$\Rightarrow \quad |f'(x)|=2a|x|$

$\Rightarrow \quad g(x)=2a|x|e^{c-ax^2}$ {even function}

$g(x)=2axe^{c-ax^2}$ when $x\ge 0$

$g'(x)=2ae^{c-ax^2}(1-2ax^2)$

$g'(x)=0$ at $x=\pm\dfrac{1}{\sqrt{2a}}$

Maximum value of $g(x)=2a\times\dfrac{1}{\sqrt{2a}}e^{c-\frac{1}{2}}=\sqrt{2a}e^{c-\frac{1}{2}}$

Comparing with $4\sqrt{e}$, we get : $a=8$ and $c=1$

Now, verify options.

8.(ABCD)

Three planes meet at two points it means they have infinitely many solutions, so

$$\begin{vmatrix}2 & 1 & 1\\ 1 & -1 & 1\\ \alpha & -1 & 3\end{vmatrix}=0 \ \Rightarrow 2(-3+1)-1(3+1)+\alpha(1+1)=0\Rightarrow\alpha=4$$

$P_1: 2x+y+z=1$

$P_2: x-y+z=2$

$P_3: 4x-y+3z=5$

P on XOY plane $=(1,-1,0)$

(Which can be obtained by putting $z=0$ in any two of the given planes.)

Q on YOZ plane $=\left(0,-\frac{1}{2},\frac{3}{2}\right)$

(Which can be obtained by putting $x=0$ in any two of the given planes.)

$\therefore$ Straight line perpendicular to plane P_3 passing through P is:

$\frac{x-1}{4}=\frac{y+1}{-1}=\frac{z}{3}$; $\quad \overrightarrow{PQ}=\hat{i}-\frac{1}{2}\hat{j}-\frac{3}{2}\hat{k}$

Projection of $\overrightarrow{PQ}$ on x-axis. $\Rightarrow \left|\frac{OP.\hat{i}}{|\hat{i}|}\right|=1$

Centroid of ΔOPQ is $\left(\frac{1}{3},-\frac{1}{2},\frac{1}{2}\right)$

9.(144) $\det.\left(A^2+A\right)=\det.\left[\left(P^{-1}DP\right)\left(P^{-1}DP\right)+P^{-1}DP\right]$

Or $\det.\left(P^{-1}D^2P+P^{-1}DP\right)$

Or $\det.\left(P^{-1}\left(D^2+D\right)P\right)$

Or $\det.\left(P^{-1}P\right).\det.\left(D^2+D\right)$

Or. $\det.(I).\det.(D).\det.(D+I)$

Or $1^3(1.2.3).(2.3.4)=6.24=144$

10.(5) When all the 5 girls are together, we treat all the 5 girls as a single girl, then 5 boys and a single girl, can stand in a queue in $6!$ ways, therefore, $n=(5!)(6!)$

When exactly four girls are together, then we can choose 4 girls in ${}^5C_4$ ways and make them together in $(6!)(4!)$ ways.

If we want exactly four girls together, then fifth girl can be arranged in 5 ways.

Thus, $m=\left({}^5C_4\right)(6!)(4!)(5)=5n \Rightarrow \frac{m}{n}=5$

11.(41) Let $P(E_1)=a, P(E_2)=b$ and $P(E_3)=c$

$3a(1-b)(1-c)=(1-a)b(1-c)=9(1-a)(1-b)c=3(1-a)(1-b)(1-c)$

$\frac{3a}{1-a}=\frac{b}{1-b}=\frac{9c}{1-c}=3 \Rightarrow a=\frac{1}{2}, b=\frac{3}{4}, c=\frac{1}{4}$

Now, $\begin{vmatrix}1/2 & 3/4 & 1/4\\ 3/4 & 1/4 & 1/2\\ 1/4 & 1/2 & 3/4\end{vmatrix}=\frac{1}{64}\begin{vmatrix}2 & 3 & 1\\ 3 & 1 & 2\\ 1 & 2 & 3\end{vmatrix}=-\frac{9}{32}$

$\therefore \quad \frac{a}{b}=\frac{9}{32} \quad \Rightarrow \quad a+b=41$

12.(101) $T_1 = \sin^{-1}\left(\frac{1}{\sqrt{10}}\right) = \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1} 2 - \tan^{-1} 1$

$T_2 = \sin^{-1}\left(\frac{1}{\sqrt{50}}\right) = \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1} 3 - \tan^{-1} 2$

$T_{10} = \tan^{-1} 101 - \tan^{-1} 100$

Sum $= \tan^{-1} 101 - \tan^{-1} 1 = \tan^{-1}\left(\frac{100}{102}\right) = \tan^{-1}\left(\frac{50}{51}\right)$

$\therefore\ p + q = 50 + 51 = 101$

13.(6) $I = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \sqrt{\frac{1-x}{1+x}} \sin^{-1} x dx = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{1-x}{\sqrt{1-x^2}} \sin^{-1} x dx$

Put $x = \sin\theta$

$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (1 - \sin\theta)\theta d\theta = \underbrace{\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \theta d\theta}_{odd, \therefore 0} - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \theta \sin\theta d\theta = -\int_{0}^{\frac{\pi}{3}} \theta \sin\theta d\theta$

$I = -2\left[-\theta\cos\theta\Big|_0^{\pi/3} + \int_0^{\pi\backslash 3} \cos\theta d\theta\right]$

$I = -2\left[\frac{-\pi}{6} + \frac{\sqrt{3}}{2}\right] = \frac{\pi}{3} - \sqrt{3} = \frac{\pi}{M} - \sqrt{N}$

$\therefore\ (M + N) = 6$

14.(5) $p_1 = \frac{\left|12 - \left(3\hat{i} - 5\hat{j} + 8\hat{k}\right).\left(2\hat{i} + 2\hat{j} - \hat{k}\right)\right|}{\left|2\hat{i} + 2\hat{j} - \hat{k}\right|} = \left|\frac{12 - (6 - 10 - 8)}{\sqrt{4 + 4 + 1}}\right| = \frac{24}{3} = 8$

Similarly, $p_2 = \frac{1}{3}\left|12 - (4 - 82 - 21)\right| = \frac{111}{3} = 37 \quad \Rightarrow \quad p_2 - 4p_1 = 5$

15. (A) $\frac{x^2}{16} + \frac{y^2}{9} = 1$ centre$(1,2)$

$X = x - 1, Y = y - 2$

$e = \frac{\sqrt{7}}{4}$, focus $\left(\sqrt{7} + 1, 2\right)$

Image of focus $= \left(-4, \sqrt{7} + 7\right)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ \sqrt{7}+1 & 2 & 1 \\ -4 & \sqrt{7}+7 & 1 \end{vmatrix}; \qquad \text{Area} = \frac{1}{2}\left(\frac{5\sqrt{7}+7}{2}\right)$$

16.(B) Do as above.

17-18 17.(D) 18.(B)

$$P^2 = P.P = \begin{bmatrix} \cos(2\pi/9) & \sin(2\pi/9) \\ -\sin(2\pi/9) & \cos(2\pi/9) \end{bmatrix}$$

$$\therefore \; P^n = \begin{bmatrix} \cos(n\pi/9) & \sin(n\pi/9) \\ -\sin(n\pi/9) & \cos(n\pi/9) \end{bmatrix}$$

$$\alpha P^6 + \beta P^3 + \gamma I = \alpha\begin{bmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -\frac{1}{2} \end{bmatrix} + \beta\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & \frac{1}{2} \end{bmatrix} + \gamma\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\frac{-\alpha}{2} + \frac{\beta}{2} + \gamma = 0 \text{ and } \frac{\sqrt{3}}{2}(\alpha+\beta) = 0 \Rightarrow \beta = -\alpha$$

$$\therefore \; \gamma = \alpha \quad \therefore \; (II)P; (III)P \text{ and } (IV)Q$$

$$\therefore \; \gamma = \alpha \quad \Rightarrow \quad f(x) = 0 \; \forall x \quad \Rightarrow \quad \sum_{k=1}^{9} f(k) = 0$$